

Game Theory: Applications to Banking and Finance

Case studies Chapter 4: Nash equilibrium - Part II

Jérôme MATHIS

Exercise: Currency speculative attack

There are two currency traders, $i = 1, 2$, possessing respectively T_1 and $T_2 \text{ €}$ (domestic currency), with $0 < T_1 \leq T_2$. The domestic central bank possesses $R \text{ €}$ in foreign currencies. By selling a large amount of the domestic currency, the traders make the central bank buying it in order to maintain a currency peg. A 50% devaluation of the domestic currency occurs whenever the central bank has spent all its reserves. If the traders succeed in causing a devaluation, they make a 50% return (in domestic currency) on the total amount of exchanged currencies. We assume that the players use pure strategies only.

For each trader let A and N denote the actions, respectively, “attack” and “not attack”. For trader i , $i \in \{1, 2\}$, to attack consists in using the total amount of his domestic currency T_i . An attack succeeds if and only if the sum of the amount used by the two traders is higher or equal to the central bank’s reserves R . A failing attack gives zero revenue. A succeeding attack gives to each trader 50% revenue of the amount he used for attacking. Any attack generates some fixed transaction costs of $c \text{ €}$, with $0 < c < \frac{T_1}{2}$, paid by each raider at the end of the attack. Trader’s payoff writes as the revenue minus the cost associated with the corresponding outcome. For instance, not attacking costs zero and yields zero, so the payoff is zero.

Part A: Traders move simultaneously.

1. Assume the central bank has enough reserves to block any attack. Formally, this hypothesis writes as: $R > T_1 + T_2$.
 - (a) Give a representation for the corresponding game.
 - (b) Find all (pure strategy) Nash equilibrium.
 - (c) Explain each previous equilibrium using one or two sentences.
2. Assume the central bank has not enough reserves to block neither collective nor individual attack. Formally, this hypothesis writes as: $R \leq T_1$.

- (a) Give a representation for the corresponding game.
 - (b) Find all (pure strategy) Nash equilibrium.
 - (c) Explain each previous equilibrium using one or two sentences.
3. Assume the central bank's reserves are sufficient to block any individual attack but are not sufficient to block a collective attack.
- (a) Write this assumption formally.
 - (b) Give a representation for the corresponding game.
 - (c) Find all (pure strategy) Nash equilibrium.
 - (d) Explain each previous equilibrium using one or two sentences.
4. Now assume there are n traders, $n \geq 2$. Suppose an attack succeeds only if all traders attack altogether. Let T_i , $i \in \{1, 2, \dots, n\}$ denotes the trader i 's amount of domestic currency. Assume $T_1 \leq T_2 \leq \dots \leq T_n$.
- (a) Find all (pure strategy) Nash equilibrium. (No game representation is required.)
5. The currency market regulators would like to discourage traders from attacking. Assume they can institute a tax t that increases the fixed transaction costs from c to $c + t$.
- (a) Give the smaller amount t that eliminates the speculative attack at equilibrium. To simplify the analysis we assume that any trader who is indifferent between attacking and not to attack will choose not to attack.

Part B: Traders move sequentially.

Let us come back to the case of two traders. Assume the central bank's reserves are sufficient to block any individual attack but are not sufficient to block a collective attack.

1. (a) Give a representation for the corresponding game.
- (b) Give the solution of the game using backward induction.
- (c) Compare your result with the one of question A.3.